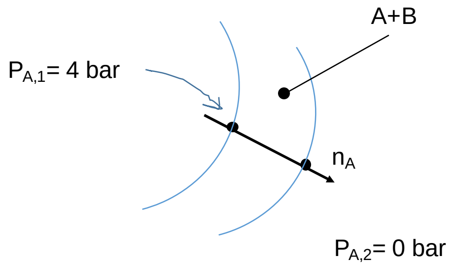
Introduction to Transport Phenomena:
Solutions to Exercises Recap Module 4 and 5

Solution to Exercise 1



Assumptions :

- D is bigger than L, thus we can consider the flux being in one dimension (it is also the only case we know how to solve ;))
- The pressure of helium outside the chamber is negligible
- Helium behaves as an ideal gas (our usual assumption)

We are asked to calculate the leakage rate. We solve this problem very similarly to 4.2. We are just going to use the absolute flux notation instead of the diffusion flux notation to keep it more general.

$$J_A = Aj_A = AD_{AB}\frac{(C_{A,1} - C_{A,2})}{L} = 4.52 \cdot 10^{-15}\ \frac{kmol}{s} = 4.52 \cdot 10^{-12}\ \frac{mol}{s}$$

Where:

$$C_{A,2} = 0$$

$$C_{A,1} = Sp_{A,1}\ =\ 1.8 \cdot 10^{-3}\ \frac{kmol}{m^3}$$

$$\frac{d(p_A)}{dt} = -\frac{J_A RT}{V} = ARTD_{AB}\frac{Sp_{A,1}}{LV} = -2.63 \times 10^{-6}\frac{Pa}{s} = -2.63 \times 10^{-9}\frac{bar}{s}$$

$$A = 4\pi\left(\frac{D}{2}\right)^2 = 0.1256\, m^2\,;\ V = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = 4.187 \cdot 10^{-3}\ m^3$$

As for 4.2 the negative sign is there because it is a leakage rate, the pressure inside the spherical container decreases over time.

Solution to Exercise 2

Because the saturation is small, we can assume that we are at steady-state and there is minimal or no variation of flux during this 3 minutes (we cannot make this assumption afterwards). Thus we can write :

$$n_A = \frac{moles\ of\ water\ evaporated}{Area \cdot time}$$

$$moles\ of\ water\ evaporated\ = C_A \cdot V$$

where $C_A$is the concentration of water in the air+water gas mixture above the liquid and V is the volume occupied by such mixture, which corresponds to 19.2L-0.8L=18.4L

$$C_A = \ x_{water\ at\ 5\%\ saturation} \cdot C_{mixture}$$

As done previously, we consider that the $C_{mixture} = C_{"pure"air}$ because we have only 5% of water.

Thus : $moles\ of\ water\ evaporated\ = x_{water\ at\ 5\%\ saturation} \cdot C_{"pure"air} \cdot V$

And

$$n_A = \frac{x_{water\ at\ 5\%\ saturation} \cdot C_{mixture} \cdot V}{Area \cdot time} = \frac{0.05 \cdot \frac{3.2\ kPa}{101\ kPa} \cdot \frac{1mol}{24.4L} \cdot 18.4L}{(150\ cm^2)(180\ s)} = 4.4 \times 10^{-8} \frac{mol}{cm^2 \cdot s}$$

In the question it is not specified if you need to determine the local mass transfer coefficient in terms of concentration or pressure (Module 4, slide 42).It will be for the exam, but let's decide to express the mass transfer coefficient in terms of concentrations. The flux is from the liquid/gas interface to the gas bulk

$$n_A = k_{c,loc}(C_{A,0} - C_{A,bulk})$$

At the interface, liquid and gas phase are in equilibrium, the partial pressure of water corresponds to the vapor pressure, thus

$$C_{A,0} = \ x_{A,0} \cdot C_{mixture} = \frac{Psat}{Ptot} \cdot C_{pure\ air} = \frac{3.2kPa}{101kPa} \cdot \frac{1mol}{24.4L}$$

as just seen $C_{A,bulk} = 0.05 \cdot \frac{3.2\ kPa}{101\ kPa} \cdot \frac{1mol}{24.4L}$

(due to the only 5% saturation, you could also do the assuption that $C_{A,bulk} = 0$)

So :

$$k_{c,loc} = \frac{n_A}{(C_{A,0} - C_{A,bulk})} = \frac{4.4 \times 10^{-8} \frac{mol}{cm^2 \cdot s}}{\frac{3.2kPa}{101kPa} \cdot \frac{1mol}{24.4L} - 0.05 \cdot \frac{3.2\ kPa}{101\ kPa} \cdot \frac{1mol}{24.4L}} =$$

$$= \frac{4.4 \times 10^{-8} \frac{mol}{cm^2 \cdot s}}{0.95 \cdot \frac{3.2\ kPa}{101\ kPa} \cdot \frac{1 mol}{24.4 \times 10^3 cm^3}} =$$

$$= 3.4 \times 10^{-2} \frac{cm}{s} = 3.4 \times 10^{-4} \frac{m}{s}$$

If we were to write the local mass transfer in terms of pressures, we would have done :

$$n_A = k_{p,loc}(P_{A,0} - P_{A,bulk})$$

$$P_{A,0} = 3.2\ kPa$$

$$P_{A,bulk} = 0.05 \cdot 3.2\ kPa$$

$$k_{p,loc} = \frac{n_A}{P_{A,0} - P_{A,bulk}} = \frac{4.4 \times 10^{-8} \frac{mol}{cm^2 \cdot s}}{3.04 kPa} = 1.48 \times 10^{-2} \frac{mol}{m^2 \cdot s \cdot bar}$$

We are then asked to calculate how long it take to reach 90% saturation. In this case we are going from dry air to 90% saturation. We have no idea of time, so we can can be more rigorous and use the more general time-dependent expression :

$$\frac{d(C_A V)}{dt} = A n_A = A k_{c,loc}\left(C_{A,0} - C_{A,bulk}(t)\right)$$

$$\frac{dC_A}{dt} = -\frac{A k_{c,loc}}{V}\left(C_{A,bulk}(t) - C_{A,0}\right)$$

If we consider that we start from dry air so $C_{A,bulk}(0) = 0$ and

We obtain $$\frac{C_{A,bulk,90\%sat}}{C_{A,0}} = 1 - e^{\frac{-A k_{c,loc}}{V} t}$$

Rearranging :

$$t = -\frac{V}{A k_{c,loc}} \ln\left(1 - \frac{C_{A,bulk,90\%sat}}{C_{A,0}}\right) = -\frac{V}{A k_{c,loc}} \ln(1 - 0.9) =$$

$$-\frac{18.4 \times 10^3\ cm^3}{(150\ cm^2)(3.4 \times \frac{10^{-2} cm}{s})} \ln(1 - 0.9) = 8300 s = 2.3 h$$

If we had assumed the flux constant over time and thus written

$$time = \frac{moles\ of\ water\ evaporated}{Area \cdot n_A}$$

$$moles\ of\ water\ evaporated\ = C_A \cdot V = \mathbf{0.90} \cdot \frac{3.2\ kPa}{101\ kPa} \cdot \frac{1mol}{24.4L} \cdot 18.4L$$

$$time = \frac{0.90 \cdot \frac{3.2\ kPa}{101\ kPa} \cdot \frac{1mol}{24.4L} \cdot 18.4L}{(150\ cm^2)(4.4 \times 10^{-8} \frac{mol}{cm^2 \cdot s})} = 3258\text{s} = 54\ \text{min}$$

Solution 3

$$\tau_{yx} = \frac{F_{shear}}{Area} = \frac{2N}{1m^2} = 2\ N \cdot m^{-2}$$

$$\tau_{yx} = -\mu \frac{dv_x}{dy} \qquad \Rightarrow \qquad \mu = \frac{-\tau_{yx}(dy)}{dv_x} = \frac{-2 \times (0.025 \times 10^{-3})}{0 - 60 \times 10^{-2}} = 8.3 \times 10^{-5}\ Pa \cdot s$$

Solution 4

The force balance on the system in +x direction:

$$\vec{F}_{app} = -\vec{F}_{shear}$$

As both forces act in the same axis, their values are equal.

$$F_{app} = F_{shear}$$

The shear stress on the shaft is given by:

$$\tau_{yx} = -\mu \frac{dv_x}{dy} = -0.65 \cdot \frac{0 - 3\frac{m}{s}}{0.3 \times 10^{-3} m} = 6.5 \times 10^3\ N \cdot m^{-2}$$

And

$$F_{shear} = \tau_{yx}\ \times\ Area$$

$$Area =\ 2\pi rh = 2\pi \times 12.5 \times 10^{-3} \times 0.5 = 39.25 \times 10^{-3}\ m^2$$

So, we have:

$$F_{shear} = \tau_{yx}\ \times\ Area = 6.5 \times 10^3\ \times 39.25 \times 10^{-3} = 255.125\ N$$

$$F_{app} = F_{shear} = 255.125\ N$$

****